Conceptual questions

26. She should not modify the procedure. Both the dart and the gorilla in the tree start falling at the same instant, and in a time \( t \) they each fall the same vertical distance \( y = n X t^2/2 \), where \( n X \) is the free-fall acceleration at the planet X. Thus the projectile inevitably will hit the gorilla at some point along the gorilla's downward path. The fact that \( n X > g \) is of no concern.

27. Assume that the initial speed \( v \) of the ball is prescribed and only the angle \( \theta \) at which the ball is kicked may vary. The vertical component of the initial velocity is \( v_y = vsin\theta \). To find the time \( t \) it takes for the ball to return to the ground, use the equation \( y = v_y t - gt^2/2 \) with \( y = 0 \) (ground level): \( vsin\theta - gt^2/2 \). There are two solutions, \( t = 0 \) (which simply corresponds to the initial point), and \( t = 2vsin\theta/g \), which is the result we sought. The greater is the angle \( \theta \), the greater is \( sin\theta \), and the greater is the time of flight.

28. The initial velocity is imparted to the ball by the face of the iron. Assume that at the instant of a strike the velocity of the club's end is horizontal and has the magnitude \( v \). If \( \theta \) is the angle of the iron's face with the shaft of the club, then the normal speed of the face is \( vcos\theta \). It is just the initial speed of the ball. Therefore, the vertical component of the initial velocity of the ball is \( vsin\theta cos\theta \) or \( (1/2)vsin2\theta \), while the horizontal component is \( vcos^2\theta \). As the angle of the face of the iron increases, so does the angle relative to the ground. When \( \theta \) increases from 0° through 45°, the ball starts off with greater initial velocity, and goes higher. When \( \theta \) increases further from 45° through 90°, the initial velocity decreases, and the ball goes lower. At the same time, as \( \theta \) increases from 0° through 90°, the horizontal range of the ball constantly gets shorter.

29. The center of mass will be located closer to the softball, because it has more mass than the tennis ball. Therefore, point A is nearest the center of mass, and this point is the most likely to follow a parabolic path.

30. The object's center of mass is the only point that will follow a parabolic path while this object is projected at an angle to the horizontal. To determine the CM of the uniform L-shaped carpenter's square, we can consider this object as two rectangles (see figure). The CM of each rectangle lies at the intersection of its diagonals. In its turn, the CM of the object lies on the line joining the CMs of the two rectangles. And it is precisely point C that is most likely nearest the object's center of mass.

Exercises

8. Let \( v = 2 \) m/s be the person's speed and \( r = 2 \) m be the radius of the merry-go-round. The centripetal acceleration experienced by the person is \( a = v^2/r = 2 \) m/s². Since the mass of the person is \( m = 50 \) kg, the centripetal force is \( F = ma = 100 \) N. It is roughly one fifth of the person's weight \( W = mg = 490 \) N.
9. The period of orbital motion expressed in seconds is \[ T = 365 \text{ days} \left( \frac{24 \text{ h}}{1 \text{ day}} \right) \left( \frac{3600 \text{ s}}{1 \text{ h}} \right) = 3.15 \cdot 10^7 \text{ s}. \]

If \( r = 1.5 \cdot 10^{11} \text{ m} \) is the radius of the orbit, then the distance covered by the Earth in one year is \( C = 2\pi r = 9.42 \cdot 10^{11} \text{ m} \). The average centripetal acceleration is \( a = \frac{v^2}{r} \), where \( v \) is the average orbital speed given by \( v = \frac{C}{T} = 2.99 \cdot 10^4 \text{ m/s} \). Therefore, \( a = 5.96 \cdot 10^{-3} \text{ m/s}^2 \) (negligible quantity as compared with \( g = 9.8 \text{ m/s}^2 \)). For the Earth’s mass \( m = 6 \cdot 10^{24} \text{ kg} \) we obtain the average centripetal force \( F = ma = 3.56 \cdot 10^{22} \text{ N} \).

10. Similar to 9. In this the given parameters are: \( T = 27 \text{ days} = 2.33 \cdot 10^6 \text{ s} \), \( r = 3.8 \cdot 10^{22} \text{ m} \), and \( m = 7.4 \cdot 10^{22} \text{ kg} \). Therefore, the acceleration of the Moon is \( a = \frac{v^2}{r} = 4\pi^2 r/T^2 = 2.74 \cdot 10^{-3} \text{ m/s}^2 \). The force between the Earth and the Moon is \( F = ma = 2.03 \cdot 10^{20} \text{ N} \).

15. The duration of the ball’s flight is completely determined by the vertical component of the initial velocity. In the equation of motion \( y = v_y t - \frac{1}{2} gt^2 \) we take \( y \) to be equal to zero at the time of landing, whence \( t = \frac{2v_y}{g} = 2.10/9.8 = 2.04 \text{ (s)} \). The horizontal range is \( R = v_xt = 20 \cdot 2.04 = 40.8 \text{ (m)} \).

16. Similar to 15. In this case we obtain \( t = \frac{2v_y}{g} = 2.30/9.8 = 6.12 \text{ (s)} \), and \( R = v_xt = 5 \cdot 6.12 = 30.6 \text{ (m)} \).

**Problem**

13. The ball falls without an initial vertical speed. Therefore, in a time \( t = 3 \text{ s} \) it gains the vertical speed \( v_y = -gt = -29.4 \text{ (m/s)} \). The horizontal speed remains unchanged: \( v_x = 40 \text{ m/s} \). Therefore, the speed with which the ball will hit the ground is \( v = \sqrt{v_x^2 + v_y^2} = 49.6 \text{ m/s} \).